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Auction-theoretic aspects of cross-border auctions

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Contents

1. Abstract

Cross-border renewable energy auctions are a topic of growing interest for policymakers, but remain under-analysed. In cross-border auctions, projects located outside of the auction-conducting country can participate and compete for support. There are numerous options for designing cross-border auctions for renewable energy support. This paper examines how opening auctions to projects in other countries influences both the allocative efficiency (i.e., projects with the lowest generation costs are awarded) and the resulting award prices.

A conceptual categorization is developed with three distinct types of cross-border auctions, each with different degrees of openness and implications for auction outcomes. The types are: Joint Auctions, where two countries implement a common auction scheme, open to projects from both countries, mutually opened auctions ("Mutual Auctions"), a scenario in which both countries open their auction schemes, and unilaterally opened auctions ("Unilateral Auctions"), where both countries conduct auctions but only one country opens its support scheme to foreign projects. Furthermore, we evaluate the outcomes of Separate Auctions, in which two countries conduct their auctions independently, while only domestic projects are allowed to participate.

Auction-theoretic modelling shows that Joint Auctions can achieve both allocative efficiency and moderate award prices. However, a complex implementation process and necessary bi-lateral coordination might make this option difficult to realise. Sequential Mutual Auctions, i.e., when the open auctions are conducted one after another and with enough time in between the auctions and not within a very short time frame, lead to similar outcomes, but with less administrative effort, since both participating countries can choose their own auction design. The remaining design choices all show a low probability of allocative efficiency and might lead to higher awarded prices. More generally, the analysis shows that parallel auctions (where project developers must choose in which auction they want to participate and cannot participate in both) tend to decrease the efficiency of a support scheme.

2. Introduction

By the end of 2018, almost 100 countries have conducted auctions for the support of electricity from renewable energy sources (RES). There are multiple different options how to design auctions, which have been collected and analysed (among others) in Mora et al. (2017). A new facet in this field is the implementation of so-called cross-border auctions, i.e., auctions that are held not only for projects situated in the auction-conducting country, but also for projects located in a foreign country. Until now only one cooperation of this kind has been conducted, namely two cross-border pilot auctions for PV in Denmark and Germany. Both Denmark and Germany conducted a cross-border auction, while Germany opened all of the 50 MW tender volume for Danish projects, whereas Denmark only opened $2,4$ MW of the 20 MW tender volume for German projects. Due to the lower prices in the German opened auction compared to the national German tenders, this auction can be considered a success in terms of lower support costs (von Blücher et al., 2019). This is one of the reasons why the pilot auction will not be be the only cross-border auction in the future. Furthermore, many EU Member States, e.g. Germany and Hungary, were obliged by the EU Commission to perform cross-border auctions (European Commission, 2014). In addition, the recently introduced revised Renewable Energy Directive (RED II) encourages countries to open at least 5% of their annual volume of auctions for RES to the participation of projects from other countries in the future (European Commission, 2018). It is therefore vital to understand the underlying theoretical framework in order to conduct cross-border auctions whose outcomes achieve the goals in the best possible way.

In this report, we examine three different ways to design cross-border auctions. Each scenario represents a different level of openness between two countries. So far, countries have conducted national auctions, only open to projects in their own country. These two *Separate Auctions* serve as a benchmark case in our analysis. A possibility of implementing cross-border auctions is the opening of one of these Separate Auctions for bidders with projects in the other country, which we call a *Unilateral Auction*. Further openness is achieved when both countries open their auctions to projects from the other country. Then bidders from both countries can decide whether they want to participate in their original country or the other (e.g. the German-Danish case). This is called a *Mutual Auction*. Complete openness is guaranteed by a *Joint Auction* conducted by both countries, which is also explicitly mentioned as a possibility by European Commission (2018). To achieve this, the countries have to decide upon one auction design. Therefore, a Joint Auction can be understood as the hardest auction format to implement. In all other scenarios, the countries decide on the auction design on their own and thus have more freedom, even if they decide to open the auction for other nationalities. All formats are auction-theoretically analysed and their outcome is compared regarding the expected rent for the auctioneer, i.e., the prices the auctioneer will have to pay (this is also often referred to as support cost efficiency), and the expected efficiency, i.e., if this format guarantees that only the bidders with the lowest costs of producing energy are awarded (generation cost/allocative efficiency). For simplicity, we assume that a country can only open their scheme completely, and not only for a percentage of the total auction volume, like this is the case in the Danish cross-border auction. We will refer to auction volume reserved for domestic projects as an outside option for those bidders, since they can compete in both auctions, the cross-border scheme as well as the country-specific one.

The rest of the report is structured as follows. In Section 3 we will introduce the different design possibilities for cross-border auctions and their individual characteristics. Section 4 will give an overview over the existing theoretical and practical literature on these auction types. In Section 5 we will develop our theoretical model. First we will introduce our basic model in Section 5.1, followed by the individual analysis on Joint (5.2), Separate (5.3), Mutual(5.4) and Unilateral Auctions (5.5). When bidders can decide in which auction they want to participate, we differentiate further between Simultaneous and Sequential Auctions. All analyses will examine efficiency and prices as main auction outcomes. We will compare the different formats in Section 5.6. Afterwards, we will present possible extensions of the model, which can be analysed in further work, in Section 5.7. We will conclude this report in Section 6.

3. Types of Cross-Border Auctions

In this section we want to analyse the difficulties and choices the implementation of the different cross-border scenarios entails. Our non-cross-border benchmark case, the complete separation of RES auctions between countries is the easiest form of auction design and implementation. Each country can then set its own design without having to interact with the other country. The awarded bidders will receive the support payment from the country their project is located in. Furthermore, it is clear that the support payments, i.e. in the European context the feed-in premiums, are based on the domestic electricity market price, so all bidders regardless of their project have the same basis for their calculations.

This is not the case if an auction is opened for projects in other countries. If the auctions are unilaterally or mutually opened for bidders from a different country, the first decision bidders have to face is in which country they want to participate. Since we first analyse auctions which take place at the same time and in coordination of each other, it is usually not economically feasible to participate in these auctions at the same time. One of the first question for the auctioneer as well as the bidders that arises from this, is which market price is the basis for the determination of the height of the support payments. The easiest way is to take the market price from the country or region the project is located in, as the energy produced is sold in this market. This can result in different calculations the bidders have to make while calculating their bid. For example, a bidder with a project located in country A but participating in country B must potentially bid differently for a fixed market premium than a bidder with a project located in country B.[1]

Nevertheless, it is still possible to have different design variants in the different countries, e.g. most likely regarding different prequalification requirements and penalties. In the German-Danish case, there were for example different financial prequalifications and ceiling prices in each auction (von Blücher et al., 2019). Further the two countries of course have different market characteristics apart from the auction design, e.g., in Germany farmland is largely excluded from PV development whereas in Denmark this is not the case. This is not explicitly part of the auction design, but part

[1] Another possibility is to take the market price of the country conducting the auction, but this goes hand in hand with higher administrative effort, since apart from the calculation of the support payment, also the difference between the two market prices has to be considered under a sliding feed-in premium. As this is rather complicated, its implementation is unlikely.

of the permits which need to be obtained in order to realise the project. These different country-specific framework conditions still persist, even if the auction designs are completely harmonised.

The by far most complex auction design for cross-border auctions is the Joint Auction. We will consider this as the cross-border benchmark case. The most intuitive form of a Joint Auction is when the two countries need to agree on one design, independently of whether they apply different designs in their country-specific auctions. In this case it cannot be distinguished which country was responsible for the award. Thus, first of all, a challenge might be the distribution of support payments between the two countries. One possibility is that the bidders are paid from a common budget, into which both countries have to pay. The easiest form here is to share the costs evenly among the participating countries, but of course all other forms of splitting costs is possible. The decision upon a fair cost distribution and auction characteristics might be hard for countries wanting to participate. Furthermore, an auction design which might be ideally adapted to the domestic market structure might not be appropriate for the neighbouring circumstances. Thus, compromises need to be found which can be rather challenging in the political and economic context. Nevertheless, we will show that the Joint Auction with a completely harmonised design is efficient and will lead to the lowest possible prices.

Another possibility to conduct a Joint Auction is to take into consideration the market differences and e.g. implement different pricing rules for the different countries, i.e., different remuneration schemes. This auction is similar to a Mutual Auction but with the difference that bidders are allowed to emit two bids, one for being awarded from country A and one for being awarded from country B. In the Mutual Auction bidders have to decide in which country they want to participate in the first place and can thus only place one bid. The bidders in this kind of Joint Auction can, similarly to the Mutual auction, only be awarded with one of their bids, i.e., either they fall under the remuneration scheme of country A or under the scheme of country B. A huge advantage of this system is that they do not have to pay penalties for the bid which is not awarded. This would not have been the case if they participated in two completely separate auctions, since then there was the possibility that they were awarded in both with the same project.

A satisfying pricing mechanism for this Joint Auction which allows only one bid per participant to be awarded can be hard to find. We will propose a solution in Section 5.7.2. The awarded bids here not only determine the height of the support payment, but also the country which has to pay for it, namely the country for which this bid was placed. Again, this procedure is overall efficient and the

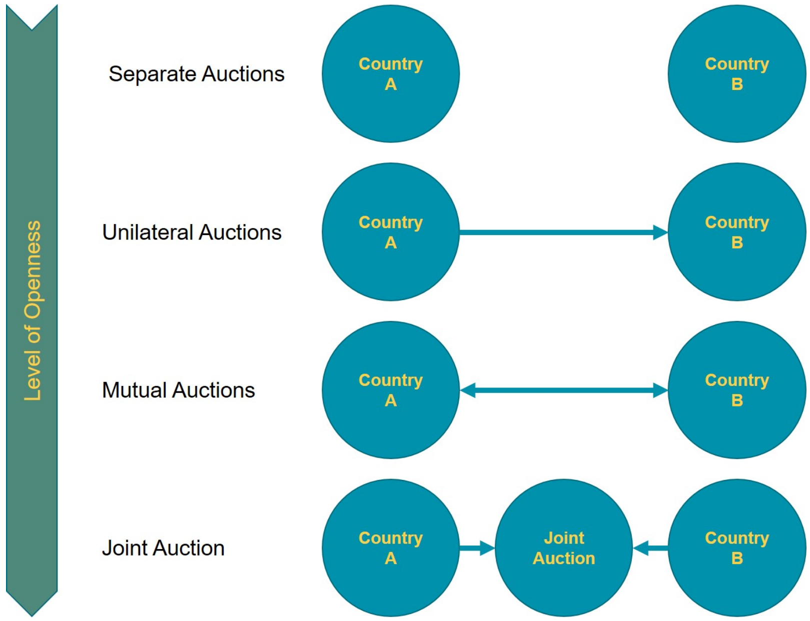

Figure 1: Overview of Types of Cross-Border Auctions

cheapest feasible outcome. Nevertheless, this can lead to major disadvantages for one country, resulting in very high costs, while the other country has very low costs. This will be discussed later in the report.

For all types of openness, we assume that either all of the auction volume is opened to foreign projects, or none. We will not consider cases where only a percentage $x\%$ of the auction volume is opened for foreign projects, while the rest of the volume is reserved for domestic plants. [2]

An overview of the different types of cross-border auctions can be found in Figure 1, where the different levels of openness of the auctions is displayed. Since there only is one auction in the joint scheme, this can be considered the scenario with the highest level of openness, and, going hand

[2] In the auction-theoretic model, this percentage scenario can easily be transformed into a scenario where there are two auctions: one for projects from both countries (and the total auction volume equal to the percentage $x\%$), and one only for bidders from the domestic country with the remaining auction volume. This assumption simplifies the analyses, as the calculation of the equilibrium participation probability is slightly different, while the bidding behaviour of the participants itself is equal in both variants. The results can thus be transferred, and need not be explained in detail in this report.

in hand with that, also the scenario with the highest level of required cooperation of the countries. The benchmark cases thus serve as boundaries of the level of openness between the countries.

4. Related Literature

In von Blücher et al. (2019) a more conceptional approach of understanding cross-border auctions is applied, examining the different design options for cross-border auctions and presenting the economic rationale for their introduction. One of the most important arguments in favour of cross-border auctions is the support cost efficiency, which can be observed in the context of the German-Danish auctions. Out of the overall 52.4 MW of auctioned volume in both auctions, only Danish projects were awarded. On the one hand, this is explained with the more favourable conditions in Denmark, e.g. the possibility to erect plants on farmland in Denmark whereas in Germany this type of location was limited. Furthermore, the Danish authorities granted much easier permits for the PV plants (Sorge, 2016) and thus preparation was easier for Danish projects, which also led to lower costs. Another factor is the market environment in both countries. In Germany, bidders had an outside option to bid in the country-specific auctions, which were conducted parallel to the cross-border auction. At that time, no Danish RES support system existed and thus, Danish bidders only had the chance of receiving support in the cross-border auctions. Subsequently, the Danish projects submitted lower bids than their German counterparts (Kahles, 2017) and thus, were in an economic sense more efficient. Furthermore, as a result, Germany did not have to pay support to the awarded bidders in most months, due to the high market values in Denmark (von Blücher et al., 2019), which is a further positive aspect from the German conducting authority's [3] point of view. We will deepen the research of von Blücher et al. (2019) in an auction-theoretical way and examine whether this apparent efficiency increase can theoretically be expected in all future cross-border auctions.

The theoretic literature on Joint Auctions is manifold, since they can be interpreted as the standard case where there are two bidder groups participating in one auction. This is for example examined in Krishna (2009). The most important finding is that an auction need not be efficient if bidder groups use different bidding strategies, i.e., if a higher bid does not necessarily correspond with higher costs, which is important when deciding on the auction design. We will use this case as a benchmark case for the cross-border auctions, in addition to the case of two Separate Auctions. The Separate Auctions themselves also can serve as the benchmark model examined in the

[3] The conducting agency is the Federal Network Agency (Bundesnetzagentur) on behalf of the German Ministry of Energy and Economics (Bundesministerium für Wirtschaft und Energie).

standard literature (e.g. Myerson, 1981) when considered by themselves. Then this corresponds to a standard IPV-model (i.e., independent private values) with a homogenous set of bidders for each auction. In this setting, each auction taken only for itself is efficient and yields the same prices and awarded bidders independently of the pricing mechanism (Revenue Equivalence Theorem; Vickrey, 1961; Myerson, 1981). This efficiency need not be the case in our setting, since we examine the groups of bidders as a whole, independently of their origin, and then it is not guaranteed that the bidders with the overall lowest costs are awarded, as we will discuss later on.

The Joint Auction is can also be connected to the literature on whether to bundle objects in one auction or to conduct Separate Auctions (Palfrey, 1983; Leszczyc and Häubl, 2010). In contrast to e.g. Leszczyc and Häubl (2010), we analyse the case where even in a Joint Auction the goods can be awarded to different bidders, i.e., can be bought from different project developers. A very practical case of centralised, i.e., Joint, auctions instead of multiple local ones can be found in Houde et al. (2017), where the market for sanitary services in Dakar (Senegal) is examined. The authors find that a centralised auction can lead up to 73% of price reductions compared to the situation where the market is not optimised. They explain this result with the increased competition due to the centralisation.

In order to be able to compare the different auction formats, we analyse two auctions being conducted at the same time but independently of each other in contrast to one single auction with a joint set of bidders. This approach can also be found in Moldovanu et al. (2006), where the authors study competing auctions compared to a centralised market place. In this paper two sellers decide whether they want to conduct one single or two separate auctions. Bidders can decide where they want to participate, but only afterwards learn their true values. One result of this paper is that there only exists an equilibrium in pure strategies in the centralised auction. We will transfer this result to the cross-border context. Another paper which deals with the auction selection problem is Delnoij and Jaegher (2018), where they conclude there is an symmetric equilibrium in mixed strategies. The focus of this paper is the design decision of the pricing mechanism, which we will not focus on, but show that this approach can be applied to mutually opened auctions as well.

A huge thread of literature related to our topic deals with the mechanism behind auctions on the internet platform ebay. Here there are many different sellers offering their goods, while bidders decide in which sales auction they participate. This is e.g. examined in Peters and Severinov (2006) with the result that in all auctions the price is identical. Further Anwar et al. (2006) describe that bidders tend to bid always in the auction with the lowest price and change auctions often. We in contrast consider a procurement auction, and bidders cannot participate in more than one auction at once. Hernando-Veciana (2005) show that if there are several auctions, in the symmetric equilibrium it is optimal for an auctioneer to choose the reserve price close to his production costs. This result can i.a. also be found in Virag (2010). For both papers the number of bidders is an important factor. We will use this result as one of the reasons why we do not focus on the optimal setting of the reserve price in this paper. A general study on the multiple bidders/multiple sellers model can be found in McAfee (1993). He shows that bidders randomize their choice which auction they participate in.

The literature on Unilateral Auctions is rather scarce. Larue et al. (2013) examine Canadian hog auctions. Here bidders from Ontario were allowed to buy Quebec hogs in an auction, but Quebec bidders did not have a chance to buy hogs from Ontario. The authors find that the increased competition was not in favour for the auctioneers since they received lower prices. Also, Gerding et al. (2008) analyse the optimal strategy for a global player who can participate in numerous local auctions, competing against local bidders. Again, in our scenario, bidders can only participate in one auction at a time. A similar setting with local bidders is considered in auctions for radio-frequency in Krishna and Rosenthalb (1996). One of the results is, that increasing the number of global bidders leads to less aggressive bidding. All of these analyses consider the possibility to buy the goods in an alternative way. This is often called an *outside option*. Outside options can have numerous different effects on auction outcomes and optimal design variables. Kirchkamp et al. (2009) show in a laboratory experiment that a first-price auction tends to generate more revenue than a second-price auction when there are outside options. This can be explained as bidders tend to be risk-averse in real life, instead of risk-neutral which is often assumed in theory. An optimal auction with outside options is charging an entry fee (Ledyard, 2007). Nevertheless, this auction may not be efficient since it prevents bidders from participation. Reiss (2008) finds that it is optimal for an auctioneer to lower the competitiveness of his auction when bidders have an outside option.

We will combine all of these different approaches into one model in order to compare the different scenarios. The outside option in our case will be that bidders can decide to participate in another auction, or if this option is not given, need to participate in the auction even though they face higher competition if they want to realise their project and receive support. This is the equivalent to the German-Danish cross-border cooperation, where German bidders had the chance to compete in the German-only auctions, whereas the Danish did not have the opportunity to participate in another auction or receive funds in any other way.

5. Theoretic Analysis

In this section, we present first theoretic analyses of different forms of cross-border auctions and their implications with regard to efficiency and awarded prices. In Section 5.1 we introduce the basic model underlying all following analyses. As a cross-border benchmark model to compare efficiency and prices, the free competition between two countries in a so-called Joint Auction is analysed in Section 5.2. Since typically, auctions are conducted - at least in the EU - on a national level, we consider this scenario of Separate Auctions in Section 5.3, which serves as our non-cross-border benchmark case. We also analyse Mutual Auctions (Section 5.4) and Unilateral Auctions (Section 5.5). For both forms, we differentiate between simultaneous auctions (Sections 5.4.1 and Sections 5.5.1) and sequential auctions (Sections 5.4.2 and Sections 5.5.2).

Auctions are analysed by game-theoretic methods. This approach is based on Vickrey (1961). A comprehensive overview and introduction to auction theory is provided by the books of Menezes and Monteiro (2005), Milgrom (2007), and Krishna (2009). The application of auctions theory to the field of renewable energy support is discussed by, e.g., Kreiss et al. (2017) and Haufe and Ehrhart (2018).

5.1. Basic Model

Consider two multi-unit procurement auctions A and B for k_A and k_B units of a homogenous good, $k_A, k_B \geq 1$. Thus, overall there are at least two goods auctioned.

There are two groups of risk-neutral bidders (firms) N_A and N_B with $n_A = |N_A|$ and $n_B = |N_B|$, $n_A, n_B \geq 1$. Moreover, $N = N_A \bigcup N_B$, $n = |N| = n_A + n_B$.

All bidders have single-unit supply, i.e., each bidder participates with one project in the auctions. The symmetric independent private values (IPV) approach applies to the two bidder sets N_A and N_B (e.g., Krishna, 2009). In Group A, each firm $i \in N_A$ has private costs x_i for supplying the good, and the firms' supply costs are independently drawn from the same distribution F_A with the density f_A and full support [4] on $[\underline{x}_A, \overline{x}_A]$. The same applies to Group B: each firm $j \in N_B$ has private costs x_j for supplying the good, and the firms' supply costs are independently drawn from the same distribution F_B with the density f_B and full support on $[\underline{x}_B, \overline{x}_B]$.

[4] Full support means that all probability mass is concentrated on this interval.

In both auctions, the auctioneers set a reserve price (maximum price) r_A in Auction A and r_B in Auction B, which the bidders are not allowed to exceed with their bids. In this report, we assume reserve prices r_A and r_B to be non-restrictive for participation, i.e., $\min\{r_A, r_B\} \geq \max\{\overline{x}_A, \overline{x}_B\}$.

The auctions are conducted as sealed-bid auctions in which each bidders i submits a bid b_i. Bids are submitted simultaneously. Let m_t denote the number of bidders who actually participate in Auction $t \in \{A, B\}$. If the reserve price does not restrict participation, $m_A + m_B = n_A + n_B$.

In the auctions the LRB-uniform-price rule applies, that is, in both auctions the lowest rejected bid determines the uniform award price p_t[5] if $m_t > k_t$. If $m_t \leq k_t$, the price is determined by the reserve price, that is, $p_t = r_t$. For bidders who participate in a single auction, an auction with LRB-uniform-pricing is incentive compatible. That is, it is a weakly dominant strategy for each bidder to bid exactly her costs x, i.e., $b = x$ (Weber, 1983).

In our analyses in the following sections, we consider cases where bidders from N_A are awarded in Auction B and vice versa. If A and B refer to different countries, Country A and Country B, this means that the awarded A-bidder will build her project in Country A and will receive the price (i.e., monetary support) from Country B (see Section 3).

The following auction-theoretic analyses also base on order statistics, which we introduce here. Consider a set N of n bidders, whose cost signals are independently drawn from distribution F with density f. The kth order statistic $X_{(k,n)}$ describes the random variable of kth lowest cost signal of all n signals (e.g., Ahsanullah et al., 2013), that is,

$$X_{(1,n)} \leq X_{(2,n)} \leq \ldots, \leq X_{(n,n)} \,.$$

The distribution function of $X_{(k,n)}$ is denoted by $F_{(k,n)}$, $1 \leq k \leq n$, and is given by[6]

$$F_{(k,n)}(x) = \sum_{i=k}^{n} \binom{n}{i} F(x)^i \big(1 - F(x)\big)^{n-i} \tag{1}$$

and the density function by

$$f_{(k,n)}(x) = \binom{n}{k} k f(x) F(x)^{k-1} \big(1 - F(x)\big)^{n-k} \,. \tag{2}$$

[5] For simplification purposes, the award prices in our study, i.e., the prices paid to the awarded bidders, correspond to a feed-in-tariff (FIT).

[6] Binomial coefficient:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

5.2. *Joint Auction*

In the Joint Auction, Auction A and Auction B are put together to one auction, in which both bidder groups N_A and N_B participate. Thus, the set of bidders in the Joint Auction is given by $N = N_A \bigcup N_B$ with $n = n_A + n_B$ and the number of auctioned goods is given by $k = k_A + k_B$. The Joint Auction serves as the reference point for the evaluation of the results of other formats in the following sections.

Let N_{eff} denote the set of bidders with the lowest costs: $N_{eff} \subset N$ and $|N_{eff}| = k$. Since it is optimal for the bidders to reveal their cost signals in their bids (Section 5.1), the k bidders with the lowest cost signals, i.e., the bidders in N_{eff}, are awarded. Hence, the auction outcome is efficient, i.e., the total demand k is met by the lowest-cost supply. The uniform price p_J in the Joint Auction is determined by the $(k+1)$th lowest cost signal $x_{(k+1,n)}$, i.e., $p_J = x_{(k+1,n)}$. That is, the auction outcome is efficient and the expected price is $E[P] = E\left[X_{(k+1,n)}\right]$.[7] The auctioneer's costs in the Joint Auction are $c_J = kx_{(k+1,n)}$. Since this cannot be determined prior to the auction, it is sensible to consider the auctioneer's expected $E[C_J] = kE[X_{(k+1,n)}]$.

The implementation in practice of a Joint Auction may be difficult and problematic and, thus, a challenge, particularly when the auction is conducted in two Countries A and B with different market characteristics and auction designs for their domestic RES auctions. Under the current rules (Sections 3 and 5.1), awarded projects of A-bidders are built in Country A and awarded projects of B-bidders in Country B. Here, the question arises, how the payments for the k awarded projects are distributed between the two countries, particularly if the number of awarded A-bidders does not match the number of demanded projects k_A in Country A. A simple rule is to allocate the payments for the best (i.e., lowest) bids from N_t to Country t until k_t is reached, $t \in \{A, B\}$. If, for example, the number of awarded A-bidders is higher than k_A and, thus, the number of awarded B-bidders in lower than k_B by the same amount, the payments for the remaining awarded A-bidders are allocated to Country B.

[7] This applies to a large set of auction formats including the pay-as-bid auction in which the (different) award prices are equal to the bids (Engelbrecht-Wiggans, 1988). This result refers to the so-called revenue equivalence theorem (Myerson, 1981; Riley and Samuelson, 1981), which states that under certain conditions any auction format that allocates the goods to the same bidders generates the same outcome including the same expected bidder profits and the same expected (average) price and auction revenue (auctioneer's costs).

5.3. Separate Auctions

In Separate Auctions, bidders from group N_A, i.e., with projects in Country A, are only allowed to enter Auction A, whereas bidders from group N_B can only enter Auction B. Since auction entry accrues no costs, all bidders will participate and thus $m_A = n_A$ and $m_B = n_B$. Therefore, for each auction, the results of the Joint Auction applies (Section 5.2). That is, the k_t bidders with the lowest costs are awarded and the price is determined by the $(k_t + 1)$th lowest cost signal $x_{(k_t+1,n_t)}$, i.e., $p = x_{(k_t+1,n_t)}$, $t \in \{A, B\}$. Thus, the expected price in Auction A is $E[P_A] = E\left[X_{(k_A+1,n_A)}\right]$ and the expected price in Auction B is $E[P_B] = E\left[X_{(k_B+1,n_B)}\right]$.

An efficient outcome is reached if and only if the $k = k_A + k_B$ bidders with the lowest costs, i.e., the bidders in the set N_{eff}, are awarded, which is met in the Joint Auction (5.2). Note that it is irrelevant for an efficient outcome in the Separate Auctions how the k bidders in N_{eff} are distributed among the two auctions A and B, i.e. which bidder participates in which auction. The only condition that has to be fulfilled is that k_A bidders in N_{eff} participate in Auction A and k_B bidders in N_{eff} participate in Auction B.

For analysing efficiency in the Separate Auctions, we consider the case of equal cost distributions in the two auctions, i.e., $F_A \equiv F_B \equiv F$. Since the distributions F_A and F_B are equal, the set N_A of the A-bidders can be modelled by n_A independent draws from F and the set N_B of the B-bidders by n_B independent draws from F. As pointed out before, an efficient outcome will be reached if exactly k_A bidders of N_{eff} are among the n_A bidders who participate in Auction A and, thus, k_B bidders of N_{eff} are among the n_B bidders who participate in Auction B. The probability that this happens is[8]

$$\frac{\binom{k}{k_A}\binom{n-k}{n_A-k_A}}{\binom{n}{n_A}}\,. \tag{3}$$

The probability in (3) includes all efficient allocations of the k bidders in N_{eff}, so that k_A of these bidders participate in Auction A and k_B of these bidders participate in Auction B. Table 1 shows the efficiency probabilities for symmetric auctions with $n_A = n_B = 25$ and $k_A = k_B$.

[8] The efficiency probability (3) can be equivalently expressed by

$$\frac{\binom{k}{k_B}\binom{n-k}{n_B-k_B}}{\binom{n}{n_B}}\,.$$

Table 1: Probability of efficient outcome in the Separate Auctions for $n_A = n_B = 25$

$k_A = k_B$	1	2	3	4	5	10	15	20
Probability	51.0%	39.1%	33.3%	29.8%	27.5%	22.7%	22.7%	27.5%

How are the prices and auctioneer's costs in the Separate Auctions compared to the Joint Auction (Section 5.2)? In an efficient outcome in the Separate Auctions, the bidder with the cost signal $x_{(k+1,n)}$ either determines the price in Auction A, i.e., $p_A = x_{(k+1,n)}$, or in Auction B, i.e., $p_B = x_{(k+1,n)}$, but not in both Auctions. That is, the prices are different in the two auctions and the higher price is $x_{(k+2,n)}$ or higher. If the bidder with $x_{(k+1,n)}$ participates in Auction A, we have $p_B > p_A = p_J = x_{(k+1,n)}$, and if the bidder with $x_{(k+1,n)}$ participates in Auction B, we have $p_A > p_B = p_J = x_{(k+1,n)}$ As a consequence the auctioneer's total costs $c_{Sim} = c_A + c_B = k_A p_A + k_B p_B$ in the Separate Auctions are higher than the auctioneer's costs c_J in the Joint Auction: $c_{Sim} > c_J$.

Now consider the prices if the outcome of the Separate Auctions is inefficient. Since there are also bidders awarded, which do not belong to N_{eff}, i.e., do not have the lowest costs und thus the lowest bids, the price in one of the two Auctions is higher than the price of the Joint Auction $x_{(k+1,n)}$. In this Auction, w.l.o.g.[9] let this be Auction A, less than k_A bidders from N_{eff} did participate. Thus, in the other Auction B, more than k_B bidders from N_{eff} did participate. Therefore not all bidders from N_{eff} are awarded, and in B a bidder with a lower bid than $x_{(k+1,n)}$ determines the price. Hence, the price in this Auction is lower than in the Joint Auction. As a consequence, in an inefficient outcome, the auctioneer's total costs can be equal or even lower than in the Joint Auction, but also higher, depending on the exact realisations of the cost signals and the actual bidder distribution over Auctions A and B.

5.4. Mutual Auctions

In Mutual Auctions, the bidders in N_A and in the bidders in N_B can participate in both auctions A and B. In the case of the Simultaneous Mutual Auctions (Sections 5.4.1), the two auctions A and B are conducted simultaneously and bidders can only participate in one of the auctions. The bid-

[9] without loss of generality

ders simultaneously decide in which auction they participate, i.e., either in Auction A or in Auction B.[10] For the case of the Sequential Mutual Auctions (Section 5.4.2), in which the two auctions A and B are conducted consecutively, we assume that the bidders are allowed to participate in both auctions. Bidders, who are not awarded in the first auction, are allowed to participate in the second auction.

5.4.1. Simultaneous Mutual Auctions

In the following we derive for the Simultaneous Mutual Auctions the game-theoretic solution in form of a symmetric mixed Bayes-Nash-equilibrium. Since the bidders simultaneously decide on the auction in which they will bid (a bidder cannot observe other bidders' decisions), a symmetric equilibrium has to be in mixed strategies, where the probability distribution of the mixed equilibrium strategy applies to participation decision. The mixed equilibrium strategy $\beta = (\beta_A, \beta_B)$ with $\beta_t(x) = (q_t, b_t)$, $t \in \{A, B\}$, consists of two components, one for the A-bidders and the other for the B-bidders, each consists of (1) the probability q_t for participating in Auction A (and thus $1 - q_t$ for Auction B) and (2) the bid b_t. Symmetry refers to both decisions: (1) All bidders in N_A participate with same probability q_A in Auction A and, thus, with probability $1 - q_A$ in Auction B. The same applies to the bidders in N_B, that is, all bidders in N_B participate with probability q_B in Auction A and with probability $1 - q_B$ in Auction B. (2) All bidders apply the same bidding strategy in form of bidding their costs x, that is, $b_t = x$, $t \in \{A, B\}$.

For determining the participation probabilities q_A and q_B, we consider a representative A-bidder with costs $x \in [\underline{x}_A, \overline{x}_A]$, who bids $b_A = x$ in the auction in which she participate and a representative B-bidder with costs $z \in [\underline{x}_B, \overline{x}_B]$, who bids $b_B = z$ in the auction in which she participates.

If each of the other $n_A - 1$ A-bidders' participation probabilities are $(q_A, 1 - q_A)$ and those of each of the n_B B-bidders are $(q_B, 1 - q_B)$ and all bidders bids their costs, the representative A-bidder's

[10] This also includes the case where auctions are not conducted at exactly the same time, but in the same time range so it is not possible to participate in both. This happened for example in the German-Danish case where only a few weeks were in between the auctions.

expected profit of bidding in Auction A is

$$\Pi_A(x, n_A, n_B, k_A, r_A, q_A, q_B) = \sum_{i=0}^{n_A-1} \sum_{j=0}^{n_B} \binom{n_A-1}{i} \binom{n_B}{j} q_A^i (1-q_A)^{n_A-1-i} q_B^j (1-q_B)^{n_B-j} I(x, k_A, r_A, i, j)\,, \tag{4}$$

$$I(k_A, r_A, i, j) = \begin{cases} \int_x^{r_A} (y-x) f_{(k_A,i,j)}(y) dy & : i+j \geq k_A \\ r_A - x & : i+j < k_A \end{cases} \tag{5}$$

and her expected profit of bidding in Auction B is

$$\Pi_A(x, n_A, n_B, k_B, r_B, 1-q_A, 1-q_B) = \sum_{i=0}^{n_A-1} \sum_{j=0}^{n_B} \binom{n_A-1}{i} \binom{n_B}{j} q_A^{n_A-1-i} (1-q_A)^{i} q_B^{n_B-j} (1-q_B)^{j} I(x, k_B, r_B, i, j)\,, \tag{6}$$

$$I(k_B, r_B, i, j) = \begin{cases} \int_x^{r_B} (y-x) f_{(k_B,i,j)}(y) dy & : i+j \geq k_B \\ r_B - x & : i+j < k_B\,, \end{cases} \tag{7}$$

where $F_{(k,i,j)}$ and $f_{(k,i,j)}$ denote the distribution function and density function of the kth order statistics (i.e., random variable of the k-lowest costs) if i cost signals are drawn from F_A and j cost signals are drawn from F_B.

Analogously, the same applies to the representative B-bidder with costs z, who bids $b_B = z$ in the auction in which she participate. Her expected profit of bidding in Auction A is

$$\Pi_B(z, n_A, n_B, k_A, r_A, q_A, q_B) = \sum_{i=0}^{n_A} \sum_{j=0}^{n_B-1} \binom{n_A}{i} \binom{n_B-1}{j} q_A^i (1-q_A)^{n_A-i} q_B^j (1-q_B)^{n_B-1-j} I(z, k_A, r_A, i, j)\,, \tag{8}$$

$$I(k_A, r_A, i, j) = \begin{cases} \int_z^{r_A} (y-z) f_{(k_A,i,j)}(y) dy & : i+j \geq k_A \\ r_A - z & : i+j < k_A \end{cases} \tag{9}$$

and her expected profit of bidding in Auction B is

$$\Pi_B(z, n_A, n_B, k_B, r_B, 1-q_A, 1-q_B) = \sum_{i=0}^{n_A} \sum_{j=0}^{n_B-1} \binom{n_A}{i} \binom{n_B-1}{j} q_A^{n_A-i} (1-q_A)^{i} q_B^{n_B-1-j} (1-q_B)^{j} I(z, k_B, r_B, i, j)\,, \tag{10}$$

$$I(k_B, r_B, i, j) = \begin{cases} \int_z^{r_B} (y-z) f_{(k_B,i,j)}(y) dy & : i+j \geq k_B \\ r_B - z & : i+j < k_B \,. \end{cases} \tag{11}$$

The equilibrium probabilities q_A and q_B are determined by

$$\begin{aligned} \Pi_A(x, n_A, n_B, k_A, r_A, q_A, q_B) &= \Pi_A(x, n_A, n_B, k_B, r_B, 1-q_A, 1-q_B)\,, \\ \Pi_B(z, n_A, n_B, k_A, r_A, q_A, q_B) &= \Pi_B(z, n_A, n_B, k_B, r_B, 1-q_A, 1-q_B)\,. \end{aligned} \tag{12}$$

In the symmetric case with $k_A = k_B$, $r_A = r_B$, $n_A = n_B$, and $F_A \equiv F_B$, by (4), (6), (8), and (10), the equilibrium conditions (12) are fulfilled with $q_A = q_B = \frac{1}{2}$. Thus, the symmetric equilibrium strategy is given by $\beta = (\beta_A, \beta_B)$ with $\beta_t(x) = (\frac{1}{2}, x)$, $t \in \{A, B\}$.

Equal Cost Distribution Functions

Any case of the Simultaneous Mutual Auction with $F_A \equiv F_B \equiv F$ can be analysed by a model of two auctions and one bidder set. The demand volumes and the reserve prices in the two auctions A and B may differ, that is, $k_A \neq k_B$ and/or $r_A \neq r_B$. Since the distributions F_A and F_B are equal, there exists a symmetric equilibrium with $q_A = q_B$, independent of n_A and n_B, which can be different. This case can be simplified by joining the two bidder sets N_A and N_B to one set $N = N_A \bigcup N_B$ with $n = n_A + n_B$, where the n bidders' signals are independently drawn from F.

The mixed equilibrium strategy $\beta(x) = (q, b)$ consists of two components. All bidders participate with same probability q in Auction A and, thus, with probability $1-q$ in Auction B. For determining the participation probabilities q, we consider a representative bidder with costs x, who bids $b = x$ in the auction in which she participates. If each of the other $n-1$ bidders' participation probabilities are $(q, 1-q)$ and all bidders bids their costs, the representative bidder's expected profit of bidding in Auction A, given by (4), (5), (8), and (9), reduces to

$$\begin{aligned} \Pi(x, k_A, r_A, q) = &\sum_{i=k_A}^{n-1} \binom{n-1}{i} q^i (1-q)^{n-1-i} \int_x^{r_A} (y-x) f_{(k_A,i)}(y) dy \\ &+ (r_A - x) \sum_{i=0}^{k_A-1} \binom{k_A-1}{i} q^i (1-q)^{n-1-i} \end{aligned} \tag{13}$$

and her expected profit if she bids in Auction B, given by (6), (7), (10), and (11), reduces to

$$\Pi(x, k_B, r_B, 1-q) = \sum_{i=k_B}^{n-1} \binom{n-1}{i} q^{n-1-i}(1-q)^i \int_x^{r_B} (y-x) f_{(k_B,i)}(y) dy \\ + (r_B - x) \sum_{i=0}^{k_B-1} \binom{k_B-1}{i} q^{n-1-i}(1-q)^i \tag{14}$$

The probability q for the mixed equilibrium strategy $\beta(x) = (q, x)$ is determined by

$$\Pi(x, k_A, r_A, q) = \Pi(x, k_B, r_B, 1-q)\,. \tag{15}$$

In the case $k_A = k_B$ and $r_A = r_B$, by (13) and (14), the equilibrium condition (15) is fulfilled with $q = \frac{1}{2}$. Thus, the symmetric equilibrium strategy is given by $\beta(x) = (\frac{1}{2}, x)$. All A-bidders and all B-bidders flip a coin to decide in which auction they will bid.

Obviously, for $k_A > k_B$ and $r = r_A = r_B$, $q > \frac{1}{2}$, since with a higher number of auctioned goods the probability of winning and thus, the expected profit in this auction rises.

The same applies for $k = k_A = k_B$ and $r_A > r_B$. To show this, we consider (13) and (14) for $q = \frac{1}{2}$. Then, the representative bidder's expected profit 13 in Auction A can we written as

$$\Pi(x, k, r_A, \frac{1}{2}) = \sum_{i=k}^{n-1} \binom{n-1}{i} \left(\frac{1}{4}\right)^{i(n-1-i)} \left(\int_x^{r_B} (y-x) f_{(k,i)}(y) dy + \int_{r_B}^{r_A} (y-x) f_{(k,i)}(y) dy \right) \\ + (r_B + (r_A - r_B) - x) \sum_{i=0}^{k-1} \binom{k-1}{i} \left(\frac{1}{4}\right)^{i(n-1-i)} \tag{16}$$

and her expected profit in Auction B can be written as

$$\Pi(x, k, r_B, \frac{1}{2}) = \sum_{i=k}^{n-1} \binom{n-1}{i} \left(\frac{1}{4}\right)^{i(n-1-i)} \int_x^{r_B} (y-x) f_{(k,i)}(y) dy \\ + (r_B - x) \sum_{i=0}^{k-1} \binom{k-1}{i} \left(\frac{1}{4}\right)^{i(n-1-i)}\,. \tag{17}$$

Since

$$\Pi(x, k, r_A, \frac{1}{2}) - \Pi(x, k, r_B, \frac{1}{2}) = \\ \sum_{i=k}^{n-1} \binom{n-1}{i} \left(\frac{1}{4}\right)^{i(n-1-i)} \int_{r_B}^{r_A} (y-x) f_{(k,i)}(y) dy + (r_A - r_B) \sum_{i=0}^{k-1} \binom{k-1}{i} \left(\frac{1}{4}\right)^{i(n-1-i)} > 0\,,$$

$q = \frac{1}{2}$ cannot be the equilibrium probability, but $q > \frac{1}{2}$.

Efficiency and Prices

An efficient outcome is reached if and only if the $k = k_A + k_B$ bidders with the lowest costs are awarded, which is met in the Joint Auction (5.2). As in the Separate Auctions (Section 5.3), efficiency does not depend on how the bidders in N_{eff} are distributed among the two auctions A and B, but only that k_A bidders in N_{eff} participate in Auction A and the remaining k_B bidders in N_{eff} participate in Auction B.

For the Mutual Auctions, we consider the symmetric case with the same equilibrium strategy $\beta(x) = (q, x)$ for all bidders in the joint set N. Then, the probability of an efficient outcome is

$$\binom{k}{k_A} q^{k_A} (1-q)^{k_B} . \tag{18}$$

The probabilities in (18) includes all efficient allocations of the k bidders in N_{eff}, so that k_A of these bidders participate in Auction A and k_B of these bidders participate in Auction B. Note that, contrary to the efficiency probabilities for the Separate Auction (Section 5.3), the efficiency probabilities for the Mutual Auctions do not depend on the number of bidders n_A and n_B in the two auctions. Table 2 shows these probabilities for symmetric auctions with $k_A = k_B$ and $q = \frac{1}{2}$.

Table 2: Probability of efficient outcome in the Mutual Auctions for $q = \frac{1}{2}$

$k_A = k_B$	1	2	3	4	5	10	25	50
Probability	50.0%	37.5%	31.3%	27.3%	24.6%	17.6%	11.2%	8.0%

To analyse and evaluate the distribution of the awarded bidders among the two bidder sets N_A and N_B in an efficient outcome, we apply the following simplifying approach. Define $\varrho = \frac{n_B}{n_A}$. In an efficient outcome, $\frac{1}{\varrho+1}(k)$ A-bidders are awarded and $\frac{\varrho}{\varrho+1}(k)$ B-bidders. This approach can be justified by an a priori view before the cost signals are drawn or by considering the average in a long-run view. For example, if $n_A = n_B$, in an efficient outcome, we expect the awarded bidders to be distributed evenly between A and B, i.e., half of the awarded bidders are from N_A and the other half from N_B.

For the prices in the Mutual Auctions the same applies as for the Separate Auctions (Section 5.3). In an efficient outcome in the Mutual Auctions, the bidder with the cost signal $x_{(k+1,n)}$ either determines the price in Auction A, i.e., $p_A = x_{(k+1,n)}$, or in Auction B, i.e., $p_B = x_{(k+1,n)}$, but not in both Auctions. That is, the prices are different in the two auctions and the higher price is $x_{(k+2,n)}$ or higher. If the bidder with $x_{(k+1,n)}$ participates in Auction A, we have $p_B > p_A = p_J = x_{(k+1,n)}$, and if the bidder with $x_{(k+1,n)}$ participates in Auction B, we have $p_A > p_B = p_J = x_{(k+1,n)}$ Hence, the auctioneer's total costs $c_M = c_A + c_B = k_A p_A + k_B p_B$ in the Mutual Auctions are higher than the auctioneer's costs c_J in the Joint Auction: $c_M > c_J$.

If the outcome of the Mutual Auctions is inefficient, the price in one of the two auctions is lower than $x_{(k+1,n)}$, while in the other auction, the price is higher than $x_{(k+1,n)}$. The auctioneer's total costs in the Mutual Auctions can be equal or even lower than in the Joint Auction, but also higher.

5.4.2. Sequential Mutual Auctions

In Sequential Mutual Auctions, the two auctions A and B are conducted sequentially. W.l.o.g. we assume that Auction A is conducted before Auction B. All bidders from N_A and N_B are allowed to participate with their project in both auctions. More precisely, all bidders are allowed to participate in Auction A, while only those bidders are allowed to participate in the B-Auction who either were not successful in Auction A or did not participate in the Auction A. We assume that the bids and results of Auction A are observable before Auction B is conducted.

In this sequential auction there exists a unique symmetric Bayes equilibrium in pure strategies. [11] In this equilibrium, each bidder submits a bid in Auction A, and if this bid is not awarded, the bidder will submit a bid in Auction B. Hence, the equilibrium bidding strategy $\beta(x)$ of a representative bidder (from N_A or N_B) with cost signal x consists of two components, $\beta(x) = (\beta_A(x), \beta_B(x))$, where $\beta_A(x)$ denotes the bid in Auction A and $\beta_B(x)$ the bids in Auction B. By transferring and extending the results of a sequential sales auction with one good in each auction (e.g. Krishna, 2009) to a sequential procurement auction with k_A goods in the first auctions and k_B goods in the second auction, we get the following equilibrium strategy $\beta(x) = (\beta_A(x), \beta_B(x))$ with

$$\beta_A(x) = E[X_{(k+1,n)} \,|\, X_{(k_A,n)} < x < X_{(k+1,n)}]\,, \tag{19}$$

$$\beta_B(x) = x\,. \tag{20}$$

Since the equilibrium strategy components $\beta_A(x)$ and $\beta_B(x)$ are strictly monotone, i.e., strictly increasing in x, the outcome of the sequential auction is efficient. That is, the k bidders with the

[11] This is a standard result in game theory, see e.g. Krishna (2009).

lowest cost signals are awarded. Strict monotonicity also implies that in the first auction A the k_A bidders with the lowest costs are awarded and in the second auction B the k_B bidders with the (k_A+1)-lowest costs up to the k-lowest costs. Thus, the "best" projects are awarded in Auction A.

Due this different bidding behaviour in Auction A and Auction B, the expected prices are the same in both auctions and equal to $E[X_{(k+1,n)}]$, i.e., the expected value of the $(k+1)$-lowest cost signal. This price, which reflects the overall scarcity in the joint market, is the same as in the free competition scenario in the Joint Auction (5.2): $E[P_{Seq,A}] = E[P_{Seq,B}] = E[P_J] = E[X_{(k+1,n)}]$. The same applies to the auctioneer's expected costs: $E[C_{Seq}] = E[C_{Seq,A}]+E[C_{Seq,B}] = k_A E\left[X_{(k+1,n)}\right]+ k_B E\left[X_{(k+1,n)}\right] = kE\left[X_{(k+1,n)}\right] = E[C_J]$.

Since the two equilibrium strategy components are monotone and, by (20), the bidders truthfully bid their costs in the second auction B, it is obvious that in the LRB-uniform-price auction B the price is equal to the $(k+1)$-lowest cost signal, i.e., the cost signal of the "best" bidder who is not awarded. In the first auction A, by (19), the bidders do not reveal their true costs but exaggerate their costs in their bids. The incentive for this form of "bid shading" is generated by the additional chance for an award in the subsequent Auction B. More precisely, a bidder's equilibrium bid for Auction A is equal to the expected value of the $(k+1)$-lowest cost signal under the condition that the bidder's own cost signal is between the k_A-lowest and the $(k+1)$-lowest cost signal. This exaggeration of the costs in the bids implies that the bidders with the k_A-lowest costs are awarded in Auction A and that the expected price in this auction is also equal to expected value of the $(k+1)$-lowest cost signal.

However, there is some empirical evidence that real sequential procurement auctions the price tends to decrease, i.e., the price in Auction B is higher than in Auction A (Ashenfelter, 1989; Ashenfelter and Genesove, 1992; Gallegati et al., 2011; McAfee and Vincent, 1993). Possible reasons for this phenomenon are risk aversion or myopic thinking. The latter refers to the fact that the bidders do not fully account for the additional chance in Auction B when calculating their bid for Auction A.

5.5. Unilateral Auctions

For the Unilateral Auctions, w.l.o.g. we assume that A-bidders are allowed to bid either in Auction A or in Auction B, while the B-bidders are only allowed to participate in Auction B.

5.5.1. Simultaneous Unilateral Auctions

Since in a Simultaneous Unilateral Auction the A-bidders simultaneously decide on the auction in which they will bid, a A-bidders' symmetric equilibrium strategy has to be in mixed strategies. As in the Mutual Auction (Section 5.4), the probability distribution of the mixed equilibrium strategy applies to participation decision, where q_A is an A-bidder's probability for participating in Auction A, and, thus, $1-q_A$ is an A-bidder's probability for participating in Auction B. As before, an A-bidder bids her cost signal in the auction in which she participates. That is, $\beta_A(x) = (q_A, x)$ The B-bidders' equilibrium strategy is simple because they cannot choose the auction. They participate in Auction B where they bid their cost signal. That is, $q_B = 0$ and, thus, $\beta_B(x) = (0, x)$.

For determining the participation probabilities q_A, we consider a representative A-bidder with costs $x \in [\underline{x}_A, \overline{x}_A]$. If each of the other $n_A - 1$ A-bidders' participation probabilities are $(q_A, 1 - q_A)$ and those of each of the n_B B-bidders are $(q_B, 1 - q_B)$ and all bidders bid their costs, the representative A-bidder's expected profit of bidding in Auction A is

$$\begin{aligned} &\Pi_A(x, n_A, k_A, r_A, q_A) = \\ &\sum_{i=0}^{n_A-1} \binom{n_A - 1}{i} q_A^i (1 - q_A)^{n_A-1-i} I(x, k_A, r_A, i)\,, \end{aligned} \tag{21}$$

$$I(k_A, r_A, i) = \begin{cases} \int_x^{r_A} (y - x) f_{(k_A, i)}(y) dy & : i \geq k_A \\ r_A - x & : i < k_A \end{cases} \tag{22}$$

and her expected profit of bidding in Auction B is

$$\begin{aligned} &\Pi_A(x, n_A, n_B, k_B, r_B, 1 - q_A) = \\ &\left(\sum_{i=0}^{n_A-1} \binom{n_A - 1}{i} q_A^{n_A-1-i} (1 - q_A)^i + n_B \right) I(x, k_B, r_B, i, n_B)\,, \end{aligned} \tag{23}$$

$$I(k_B, r_B, i, n_B) = \begin{cases} \int_x^{r_B} (y - x) f_{(k_B, i+n_B)}(y) dy & : i \geq k_B \\ r_A - x & : i + n_B < k_B \end{cases} \tag{24}$$

where $F_{(k,n_B+i)}$ and $f_{(k,n_B+i)}$ denote the distribution function and density function of the kth order statistics (i.e., random variable of the k-lowest costs) if i cost signals are drawn from F_A and n_B cost signals are drawn from F_B.

The equilibrium probability q_A is determined by

$$\Pi_A(x, n_A, k_A, r_A, q_A) = \Pi_A(x, n_A, n_B, k_B, r_B, 1 - q_A)\,. \tag{25}$$

That is, every A-bidder is indifferent (with respect to her expected profits) between participating in Auction A or in Auction B.

Equal Cost Distributions

The case of an equal cost distribution is given by $F_A \equiv F_B$. For simplicity, we further assume $r_A = r_B$ and $k_A = k_B$.

If $n_A \leq n_B$, we have $q_A = 1$ and, thus, $1 - q_A = 0$ because

$$\Pi_A(x, n_A, k_A, r_A, q_A) > \Pi_A(x, n_A, n_B, k_B, r_B, 1 - q_A) \tag{26}$$

for all $q_A \in [0, 1]$. That is, no A-bidder participates in Auction B because an A-bidders expected profit (21) from participating in Auction A is always higher than her expected profit (23) from participating in Auction B, independent of the other A-bidders' decision. This holds because actual number of competitors the Auction B is always higher than in Auction A. Thus, this case is equal to the case of two separate auctions (Section 5.3).

Only for $n_A > n_B$, $q_A < 0$ and, thus, $1 - q_A > 0$, i.e., the A-bidders also participate with a positive probability in Auction B. Given a fixed n_B, it follows from (21), (23), and 25 that q_A decreases in n_A. That is, the higher the number of A-bidders, the higher is the probability that they participate in Auction B. For these cases, with regard to efficiency, expected prices and costs, the argumentation and results of the Simultaneous Mutual Auctions (5.4.1) apply. That is, there is a high probability that the outcome is inefficient and that the prices and costs are higher than in efficient outcome of the Joint Auction (Section 5.2).

5.5.2. Sequential Unilateral Auctions

When analysing Sequential Unilateral Auctions, we have to distinguish between the case that Auction A is conducted before Auction B and the opposite case that Auction B is conducted before Auction A. Since B-bidders are only allowed to participate in Auction B, it is optimal for them to bid their cost signal independent of the sequence of the auctions. For the A-bidders it is optimal to participate in both auctions by bidding truthfully in the second auction and exaggerating their costs in the first auctions. This is the same bid pattern as in the Sequential Mutual Auctions (Section 5.4.2). However, the degree of exaggeration in the first auction differs from (19).

The award prices p_A and p_B depend on the sequence of the auctions.

If Auction A is conducted before Auction B, $p_B \leq x_{(k+1,n)}$. The case $p_B = x_{(k+1,n)}$ holds if and only if the outcome of the Sequential Unilateral Auction is efficient, i.e., the k bidders with the lowest costs are awarded, i.e., the bidders in set N_{eff}. In this case, in Auction A, k_A of these bidders are awarded, and in the subsequent auction B, the remaining k_B bidders.

If k_A or more A-bidders are in N_{eff}, the outcome is efficient and $p_B = x_{(k+1,n)}$ because all bidders in N_{eff} are awarded and the bidder with $x_{(k+1,n)}$ determines the price in Auction B. If in this case, the A-bidders beliefs about N_{eff} are correct, $E[P_{Uni,A}] = E\left[X_{(k+1,n)}\right]$. That is, the expected price in both auctions are equal and equal to the expected price in the Joint Auction 5.2: $E[P_{Uni,A}] = E[P_{Uni,A}] = E[P_J] = E\left[X_{(k+1,n)}\right]$. Thus, this also applies to the auctioneer's expected costs: $E[C_{Uni}] = E[C_{Uni,A}] + E[C_{Uni,B}] = k_A E\left[X_{(k+1,n)}\right] + k_B E\left[X_{(k+1,n)}\right] = kE\left[X_{(k+1,n)}\right] = E[C_J]$.

If at least one of the k_A bidders, who are awarded in Auction A, does not belong to N_{eff}, i.e., more than k_B bidders are in N_{eff}, the outcome is inefficient and $p_B < x_{(k+1,n)}$. This happens because not the bidder with $x_{(k+1,n)}$ determines the price in Auction B, but a bidder in N_{eff} with a lower cost signal than $x_{(k+1,n)}$. This case occurs if fewer than k_A A-bidders and, thus, more than k_B B-bidders are in N_{eff}. In this case, the price p_A differs from p_B. In Auction A, the price-determining A-bidder's cost signal is higher than $x_{(k+1,n)}$. Since A-bidders exaggerate their costs in their bids, $p_A > x_{(k+1,n)}$. Therefore, in this case, $p_B < x_{(k+1,n)} < p_A$.

If Auction B is conducted before Auction A, p_A is ambiguous, i.e., all cases $p_A = x_{(k,n)}$, $p_A < x_{(k,n)}$, or $p_A > x_{(k,n)}$ are possible. Moreover, the equivalence between efficiency and $p_A = x_{(k+1,n)}$ does not hold as in the opposite sequence. The Sequential Unilateral Auction is efficient if the k bidders with the lowest costs are awarded. In this case, in Auction B, k_B of these bidders are awarded, and in the subsequent auction A, the remaining k_A bidders. However, since the A-bidders and B-bidders behave differently in the B auction – the B-bidders bid truthfully, whereas the A-bidders exaggerate their costs – it is possible that in Auction B, a B-bidder, who is not N_{eff}, is awarded. As a consequence, the outcome is inefficient and $p_A < x_{(k,n)}$ because the price in Auction A is determined by a bidder with a cost signal $x < x_{(k+1,n)}$. On the other side, if more than k_B bidders are in N_{eff}, the outcome is also inefficient and $p_A > x_{(k+1,n)}$ because the price in Auction A is determined by a bidder with a cost signal $x > x_{(k+1,n)}$. Also in theses cases, the prices p_A and p_B may differ.

5.6. Comparison of the Different Auctions

In order to decide on an optimal design for cross-border auctions, it is important to compare the different auction scenarios and their individual outcomes. In Section 5.2 we showed that a Joint Auction is always efficient and yields an expected price of $E[P] = E\left[X_{(k+1,n)}\right]$ when bidders from the two countries can be assumed to have similar costs. If one conducts Separate Auctions, the probability of an efficient outcome is much smaller as calculated in Section 5.3, and if this auction is efficient, the price in one of the auctions will in all cases be higher than the price of the Joint Auction. If the auctions end inefficient, i.e., if not the k projects with the lowest costs are awarded, the outcome cannot be determined before. The overall costs for the auctioneers can be higher than, lower than or equal to the costs of the joint auction. This is because in one of the auctions the auctioneer will have to pay less than in an efficient outcome, but the other one has to pay more. Depending on this exact ratio, the overall costs can be determined. One argument in favour of Separate Auctions is the relatively easy implementation for each country, since they do not need to cooperate.

Mutual Auctions can be conducted both simultaneously and sequentially. When conducted simultaneously, the problem is the same as with Separate Auctions, since the probability for an efficient outcome is rather small for a high number of auctioned goods (Section 5.4.1) and efficiency leads to higher prices than in the ideal case of the Joint Auction. Again, if the auction outcome is inefficient, no concrete statement about the resulting prices can be made. When the Mutual Auctions are conducted sequentially, the outcome is efficient and the prices in both auctions are equal to the price of the Joint Auction (Section 5.4.2). This constitutes a real alternative for the Joint Auction. The outcomes are identical, but the Mutual Auctions leaves much more liberties for the conducting countries, since each country is responsible for only one auction independently of the design of the other. Of course, if the designs are too different, this will have effects on the prices as well. A further advantage is that it is clear which country gave the award for which bidder and thus has to pay for the support.

The Unilateral Auctions are comparable to the Separate Auctions if the cost structure and auction design is identical (or considerably similar) in both countries (Section 5.5.1). In this case, nobody will enter the auction in the foreign country and the results are those from Section 5.3. If the market characteristics or the auction design are different in both countries, again an equilibrium in mixed strategies is constituted and the result is the same as Section 5.4.1 for Simultaneous Mutual

Auctions. The Sequential Unilateral Auctions need to be distinguished into two scenarios: one, the opened auction is conducted first, and two, it is conducted second. If it is conducted first, there is a high chance that the auctions will be overall inefficient, since in the first auction two different bidding strategies are apparent: the bidders only allowed to participate in this auction will bid their true costs while the others will apply bid-shading (Section 5.5.2). Thus, depending on the cost structure of the bidders, the prices can differ in both directions. What is clear is that both auctions will in most cases not achieve the same prices.

If the opened auction is conducted second, the auctions can be efficient. This is the case if more than k_A A-bidders are in the group of the overall lowest costs projects (Section 5.5.2), as this secures that in both auctions only the bidders with the lowest costs are awarded. In this case, the price in both auctions will be equal to the price achieved in the Joint Auction. If this is not the case, i.e., if there are bidders awarded in the first auction who do not belong to the group with the lowest costs, the auction outcome is inefficient and the prices of the two auctions differ from the reference price $E\left[X_{(k+1,n)}\right]$, where the price in the first auction is higher, and the price in the second auction is lower than $E\left[X_{(k+1,n)}\right]$.

To put it in a nutshell, the Joint Auctions has a guaranteed efficient outcome and no dangers of too high prices due to unfavourable bidder structures in the different countries. Nevertheless, it is harder to conduct. An alternative would be the Sequential Mutual Auction, where the same outcome regarding awards and prices can be expected, but with more liberties for the auctioneers in their individual auction design. The other auction types can yield lower overall prices, but only together with an inefficient outcome. Furthermore, there is also a high chance that the prices will turn out to be higher than in a Joint Auction.

5.7. Extensions

In the following we discuss some extensions of the models in the previous sections.

5.7.1. Other Auction Formats

How do the results of our analyses change instead of the LRB-uniform price rule the Pay-as-bid rule or the HAB-uniform price rule are applied (HAB: highest accepted bid)? In Section 5.2, we mention the so-called revenue equivalence theorem (Myerson, 1981; Riley and Samuelson, 1981; Engelbrecht-Wiggans, 1988), which (to a certain degree) can be applied to different pricing rules in the auctions considered in the previous sections. Accordingly, the expected equilibrium outcomes under other pricing rule are considered to be the same or at least similar to theses derived under LRB-uniform pricing.

5.7.2. Systematic Cost Differences between Countries

Assume that Auction A is conducted in Country A and Auction B is conducted in Country B and that there are systematically different conditions in the two countries. These differences may be caused by differences in the monetary support systems in the two countries. Due to these differences, bidders have different costs for a similar project depending where the project is built.

W.l.o.g. we assume that the costs are higher in Country B than in Country A. We model this cost difference by an additive constant s. That is, if a bidder has costs x when she is awarded in Auction A, the bidders has costs $x+s$ when she is awarded in Auction B. As a consequence, the bidder submits a higher bid in Auction B than in Auction A. In the case of the simultaneous auctions (Sections 5.4.1 and Section 5.5.1), the different bids are $b=x$ in Auction A and $b=x+s$ in Auction B. Generally, in the Separate Auctions (Section 5.3, Mutual Auctions (Section 5.4), and Unilateral Auctions (Section 5.5), the price in Auction B is expected to be s higher than in the case of equal costs with $s=0$ considered so far. The results about efficiency and expected costs remain except for the auctioneer's costs in Auction B, which increase by $k_B s$.

The Joint Auction is a challenge because the two auction demands k_A and k_B are put together and are allocated in one auction, in which each bidder submits one bid for her project. Therefore, for the Joint Auction, we recommend that the two countries agree on one award system, so that it does not matter for the bidders whether they are awarded in Country A or B.

Nevertheless, it is possible to design a reasonable and applicable mechanism for the Joint Auction which takes the systematic cost differences between the two countries into consideration. It is obvious that the allocation procedure described in Section 5.2 cannot be applied because it does not account for the cost differences. The proposed design for the Joint Auction contains is based on the Generalized Vickrey Auction (GVA) or Vickrey-Clarke-Groves (VCG) mechanism (e.g., Ausubel and Migrom, 2006; Krishna, 2009). Each bidders submits two bids for her project. The first bid, the A-bid, applies to Country A and the second bid, the B-bid, applies to Country B. From all submitted bids, the set of all feasible combinations of bids is computed. A feasible bid combination contains (1) at maximum one bid of each bidder and (2) k_A A-bids and k_B B-bids. The winning bids are determined by the feasible bid combination that minimizes the total sum of bids.

For the pricing rule, the Vickrey rule (e.g., Ausubel and Migrom, 2006) or the pay-as-bid rule can be taken into consideration, whereas the uniform price rule (LRB or HAB) is considered to be less suited. Although the Vickrey Auction is incentive-compatible, i.e., it is a weakly dominant strategy for the bidders to reveal their true costs in their bids b_A and b_B, due to the weaknesses of this auction format, we consider the application of the pay-as-bid rule to be the better choice.

5.8. Multiple Countries

In the future, it might be considered to not only conduct cross-border auctions between two countries, but between multiple ones, e.g., the implementation of region-wide cross-border auctions can be of interest. This can for example be sensible in the Baltic Region, since the countries are relatively small and the introduction of an auction scheme can be administratively challenging. Our results can easily be extended to multiple countries. The Joint Auction and Separate Auctions will maintain their effects. For the Mutual and Unilateral Auctions, there are just a few cases added, for example when one country decides to open for all other countries while another one decides to open only for one foreign country. Nevertheless, the principle problems and underlying structures remain, i.e., the auctions will with a high probability be inefficient and the overall costs for the auctioneers, i.e., the different countries, might be much higher than with a cooperative design like the Joint or Sequential Mutual Auction.

6. Conclusion

In this paper, we have theoretically analysed the implications of various degrees of openness of cross-border auctions on the allocative efficiency and the resulting award prices. We compared the results of a Joint Auction, Separate Auctions, a Unilateral Auction and Mutual Auctions. In our approach, we assume an adequate auction design and sufficient competition in all auctions.[12]

We find that the Joint Auction is the most promising type of cross-border auction with regard to efficiency, our modelling result showing efficient allocation, as well as moderate awarded prices. Nevertheless, implementing this type of auction is quite complicated due to a high degree of cross-border integration and regulatory coordination.

Mutual Auctions can be conducted either simultaneously or sequentially. In the first case, we find that the probability of achieving an efficient outcome is rather small and the resulting prices are higher than in the Joint Auction. Sequential Mutual Auctions, on the other hand, lead to an efficient result as well as to the same prices as in the Joint Auction. This type of cross-border auction, which has already been used in the German-Danish PV auctions, can thus be a role-model for future design choices. Policymakers do not face the same difficulties as in the Joint Auctions: each country can decide on its own auction design and thus no coordination efforts are needed. In addition, each country can be responsible for the support payments awarded in its own auction, and no complex formula is needed to divide the support payments, as required in the Joint Auction. Nevertheless, as studies have shown, in reality lower costs might be expected in the first Mutual Auction compared to the second one, thus this has to be accounted for when countries establish their cross-border cooperation.

Unilateral (both the Simultaneous and Sequential cases), as well as the Separate Auctions are shown to have a relatively low probability of achieving an efficient outcome and are thus inferior to both the Joint and the (Sequential) Mutual Auctions. More generally, the analysis shows that parallel auctions (where project developers must chose in which auction they want to participate and cannot participate in both) tend to decrease the efficiency of a support scheme.

Therefore, based on our theoretical analysis, we can recommend to policymakers to consider Sequential Mutual Auctions when designing cross-border auctions. This auction type combines

[12] Note that the specific design implications of each type of auction is not the focus of this report.

the benefits of relatively straightforward implementation with the allocative efficiency of a Joint Auction.

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AURES II is a European research project on auction designs for renewable energy support (RES) in the EU Member States. The general objective of the project is to promote an effective use and efficient implementation of auctions for RES to improve the performance of electricity from renewable energy sources in Europe.

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